5.2 Diamagnetische Festkörper

Einfachster Fall: Atome, bzw. Festkörper aus Atomen mit **vollständig gefüllten e^--Schalen**
→ Erwartungswerte für Spin und Bahndrehimpuls =0
(sonst überdeckt von Para-/Ferromagnetismus)

gilt z.B. für :

- **Edelgase** (auch Edelgas-Kristalle, da Van-der-Waals-Bindung die e^--Konfiguration der freien Atome nur unwesentlich verändert)
- **Ionenkristalle** der Alkali- und Erdalkalihalogenide (z.B. NaCl)
  bilden abgeschlossene Schalen mit Edelgaskonfiguration infolge Ladungstransfer von Metall- → Halogenatomen)

Ursache: Präzessionsbewegung der einzelnen atomaren Bahnmomente
um das angelegte Feld
→ Abschwächung eines äußeren Magnetfeldes
(analog Lenzscher Regel für Induktionsströme)

weiterer möglicher Beitrag: freie e^- → *Diamagnetismus der Leitungselektronen* (s. später)

zunächst: nur Beitrag der (lokalisierten) Gitterbausteine → **gute Beschreibung für Isolatoren**

Quantitative Behandlung – nach Langevin
auf Basis des Bohrschen Atommodels:

Grundlage → Larmor-Theorem (zunächst für einzelne Atome):

Für $\vec{B} = (0, 0, B_z)$ präzediert Elektronenbahn um $\hat{z}$ mit der Larmor-Frequenz

$$\omega = \frac{eB_z}{2m} \tag{5.12}$$

Konsequenz:

Larmor-Präzession von Z Elektronen ≅ elektrischer Strom

$$I = (\text{Ladung})(\text{Umläufe/Zeit}) = -(Ze)\left(\frac{1}{2\pi}\frac{eB}{2m}\right) \tag{5.13}$$

entspricht einem magnetischen Moment (Strom × Fläche A mit Bahnradius ϱ)

$$\Rightarrow \vec{\mu} = IA\,\hat{e}_n = -\frac{Ze^2B}{4\pi m}\pi\langle\varrho^2\rangle\hat{e}_z \tag{5.14}$$

mit dem mittleren Abstandsquadrat $\langle\varrho^2\rangle = \langle x^2\rangle + \langle y^2\rangle$ des e^- $\perp$ Feldachse durch Kern.

Bei kugelsymmetrischer Ladungsverteilung gilt $\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle$ und somit für das mittlere Abstandsquadrat $\langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle$ der e^- vom Kern

$$\langle r^2\rangle = \frac{3}{2}\langle\varrho^2\rangle \tag{5.15}$$

Eingesetzt in (5.14) liefert für den Betrag des magnetischen Moments in z-Richtung

$$\mu = -\frac{Ze^2B}{6m}\langle r^2\rangle \tag{5.16}$$

Mit $M = \mu \cdot N$ und mit $\chi_{dia} \approx \chi_{dia}/(1+\chi_{dia}) = \mu_0 M/B$ folgt dann für die diamagnetische Suszeptibilität (pro Volumeneinheit) die **Langevin-Gleichung:**

$$\chi_{\text{Dia}} = \frac{\mu_0 N}{B}\cdot\mu = -\frac{\mu_0 N Ze^2}{6m}\langle r^2\rangle \tag{5.17}$$

(N: Zahl der Atome pro Volumen)

Bemerkungen:

- Berechnung von χ beschränkt sich also auf die (quantenmechanische) Berechnung von $\langle r^2\rangle$ für die e^--Verteilung des Atoms
- $\chi \propto \langle r^2\rangle \;\Rightarrow$ vor allem äußere Bahnen liefern Beitrag zu Diamagnetismus
  z.B. d-Elektronen bei den diamagnetischen Edelmetallen
  Ag, Au und Cu, sowie bei Pb
- Experiment $\pm 10\%$
  (Für Edelgase, dielektrische Festkörper (Isolatoren),
  d.h. Alkalimetalle und Halogenide in Ionenkristallen)

  Abweichungen durch Beiträge der Leitungselektronen → komplizierter
- Typische Werte für χ_{Dia}: $-10^{-4}\ldots-10^{-6}$
  (vgl: $\chi = -1$ in Supraleitern !)

- **Molsuszeptibilität** χ_M in cm^3/mol: $\chi_M = \chi/n$, mit n=mol/cm^3
  oder bezogen auf Dichte ϱ →

- **spezifische Suszeptibilität** $\chi_s \equiv \chi/\varrho$
  aus einfacher Abschätzung mit $\langle r^2\rangle \approx r_B^2$ in (5.17)

$$\chi_s \equiv \frac{\chi}{\varrho} \approx -4\times 10^{-5}\frac{\text{cm}^3}{\text{g}} \tag{5.18}$$

Tabelle 5.1: gemessene Molsuszeptibilitäten χ_M und spezifische Suszeptibilitäten χ_s [aus Kittel, *Einführung in die Festkörperphysik* (1999); S. 446 und Weissmantel, Hamann, *Grundlagen der Festkörperphysik* (1978); Tab.10.1]

	He	Ne	Ar	Kr	Xe	H_2	N_2	H_2O
χ_M [10^{-6}cm^3/Mol]	-1.9	-7.2	-19.4	-28.0	-43.0			
χ_s [10^{-5}cm^3/g]	-0.59	-0.45	-0.61			-2.5	-0.54	-0.905

- χ_{Dia} ist unabhängig von der Temperatur (s. Abb.5.3)
- rigorose quantenmechanische Betrachtung liefert gleiches Ergebnis
- Diamagnete können in externen Feldern schweben (Bzw. Feldgradienten)

Quantenmechanische Theorie des Diamagnetismus

Betrachte mononukleares System $Z = 1$

Hamilton-Operator ($\mathcal{H}\Psi = E\Psi$) eines Teilchens im Magnetfeld

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 + e\varphi \tag{5.19}$$

(m: Masse, $\vec{p}$: Impuls $\cong i\hbar\vec{\nabla}$, e: Ladung, $\vec{A}$: Vektorpotential, ($\vec{B} = \mathrm{rot}\vec{A}$),
φ: elektrostatisches Potential)

d.h. das Magnetfeld erzeugt im Hamilton-Operator die zusätzlichen Terme

$$\mathcal{H}' = \frac{ie\hbar}{2m}\left(\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}\right) + \frac{e^2}{2m}A^2 \tag{5.20}$$

Diese können für ein Atomelektron gewöhnlich als kleine Störung behandelt werden.

Für homogenes Magnetfeld in z-Richtung gilt

$$A_x = -\frac{1}{2}yB \quad A_y = \frac{1}{2}xB \quad A_z = 0 \quad . \tag{5.21}$$

Damit wird aus (5.20)

$$\mathcal{H}' = \frac{ie\hbar B}{4m}\underbrace{\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)}_{\propto L_z} + \frac{e^2B^2}{8m}(x^2 + y^2) \tag{5.22}$$

auf rechter Seite von (5.22):

- linker Term:
  proportional zu L_z (z-Komponenten des Bahndrehimpulses $\vec{L} = m\vec{r}\times\vec{v}$)
  liefert in mononuklearen Systemen nur paramagnetischen Beitrag
- rechter Term:
  Energie aus Lösung der Schrödinger-Gleichung: $E' \ll E$, d.h. "kleine Störung"

  für ein kugelsysmmetrisches System folgt aus Störungstheorie 1. Ordnung ein Beitrag

  $$E' = \frac{e^2B^2}{12m}\langle r^2\rangle \tag{5.23}$$

  Mit $E' = -\mu B$ ist das zugehörige magnetische Moment diamagnetisch

  $$\mu = -\frac{dE'}{dB} = -\frac{e^2B^2}{6m}\langle r^2\rangle \tag{5.24}$$

  in Übereinstimmung mit dem klassischen Ergebnis.

Levitation:

Kraft im inhomogenen Magnetfeld

$$\vec{F} = \vec{\mu}\nabla\vec{B} = \vec{M}V\nabla\vec{B} \tag{5.25}$$

(hier ist μ die Summe aller magn. Momente, und V das Probenvolumen)

mit $\vec{M} = \frac{\chi}{\mu_0}\vec{B}$ und $\nabla B^2 = 2\vec{B}\nabla\vec{B}$ gilt dann

$$\vec{F} = \frac{\chi}{\mu_0}V\vec{B}\nabla\vec{B} = \frac{\chi}{\mu_0}\frac{V}{2}\nabla B^2 \tag{5.26}$$

Bedingung: Kompensation durch Schwerkraft $F_{dia} = F_g = mg = \varrho V g$
(ϱ: Dichte der Probe)
ergibt

$$\nabla B^2 = 2\mu_0 g\frac{\varrho}{\chi} \tag{5.27}$$

Typische Werte für $\chi/\varrho \approx 10^{-5}\mathrm{cm}^3/\mathrm{g}$

$\Rightarrow$ erfordert vertikalen Feldgradienten $\nabla B^2 \approx 25\,\mathrm{T}^2/\mathrm{cm}$

Mit $\nabla B^2 \approx B^2/l$ und $l \approx 10\,\mathrm{cm}$ für typische Hochfeld-Spulen

$\Rightarrow$ B $\approx (10\,\mathrm{cm}\cdot 25\,\mathrm{T}^2/\mathrm{cm})^{1/2} \approx 15\,\mathrm{T}$ erforderlich !!

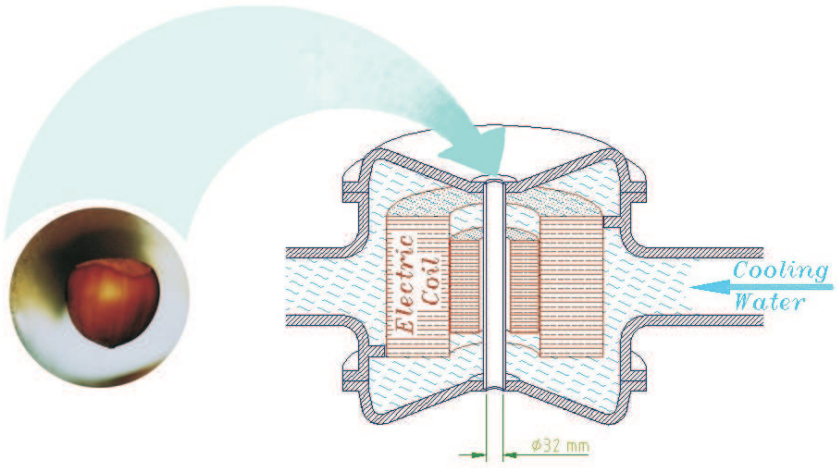

Abb. 5.5: Levitating Nuts - experimental setup: The object, in this case a hazelnut (inset), is placed in the 3.2 cm bore of a 20 T Bitter magnet. When the field in the center is about 16 T, the magnet gradients at the levitation point (near the top of the inner coil) are just right to cancel the pull of gravity at the molecular level in this manifestly nonmagneticöbject. There, the applied field is about 10 T and the nut becomes a weak magnet, having an induced field of about one gauss. This implies a surprisingly large current (about one amp) circulating in the nuts shell, but the current represents the summation of induced microscopic currents in atoms and is not dissipative. Thus, living creatures are not electrocuted when levitating. [aus Andrey Geim, *Physics Today* September 1998].

[aus http://www.sci.kun.nl/hfml/levitation-pubres.html]

FOUNDED 1905
BARKING, ESSEX

REVIVED 1995
BRADFORD ON AVON

The Church of The Latter Day Snakes

INCORPORATING THE ONE TRUE CHAPEL OF HERPETOLOGY

Professor Main,
The Physics Department,
The University of Nottingham.

14 April, 1997.

Dear Professor Main,

I and my closest associates who are good eggs in the Church of the Latter Day Snakes were very fascinated to read a reporting of your experiment with a powerful magnet and a frog in The Independent, of Saturday, 12 April, 1997. You claim that you are able to levitate a frog and even fish and plants too by means of your machine. We in the church are not scientists, we follow the spiritual path, and so do not understand this business. Does it work, or is it merely just hokum and bollocks? Please do not think I am rude when I ask this question, but you know there are many charlatans and those who sell snake oil, like in the John Wayne film.

We have a big interest in this machine if it works and I will tell you this subsequently, but first I would like to know certain answers.

(1) How big is this magnet, and can it be concealed beneath a floor, perhaps? It is important for our ideas that it can not be seen. Will it work if there is wood there? And the floor nails. Will they mess up the magnet?

(2) Does it make much noise, and if so is it a loud noise? A quiet hum would be alright of course because we have a Hammond organ.

(3) We are interested naturally in levitating bodies. Must they be naked bodies, or can they have clothings? You say they would have to be lying down but that would be alright at a pinch.

(3a) Does it hurt, and can you bring the body back down without damages, because it will be me doing the levitating. I am quite large being 22 stone weight, but my mother says I have heavy bones! No, jokings put aside, most of me is liquid I think and I am not very dense so maybe that is good for your machine.

Please answer me first these questions and then you are my friend. I must trust you first before we do business. For you, you must be interested to know that our church is very rich. We have nearly twenty five million pounds in gilt edge securities and properties in Essex and Kent, so if everything is good we want to buy your machine for one million pounds, which would be a good price.

So you know what I have in mind I will explain a little perhaps.

Our church was founded in 1905 by Alfred Muzzlewhite, but after he died it was not the same and in 1950's it all went belly-up. The good news was that all the money was still in the banks. In 1995 I am coming from Norway and I make the church go again. We have many followers in Wiltshire, but we want many more in all Britain. Funny, hah? We have all this money, and we have the One True Word to save the world, but we have to do magic tricks to get the peoples to listen! But this is why we need your machine.

I hope you don't have a problem with that. I know in our church services if we pull back the curtain in the Snake Chamber and I start to rise up from the ground and then (slowly and gently!) come back down again I will get many more to join the church, so this will we very good and explains to you my plans. It is important if we do this deal that you do not tell anyone of course. But a million pounds buys a lot of hush-hush, as James Cagney says in his film, although then for him it was dollars.

I have only one other question. I have been reading in The Times about the Natural Law Party and they also do levitating. I wonder have they been in touches with you as well? When we are partners perhaps you can tell me. Please do not sell them a machine. They are very bonkers and very dangerous to boot. And also. It says in the newspaper that you will be using the machine to test chemicals and systems in space. You must not be doing this! Science. PAH!! You have a gift. Many are called but few are chosen. Your machine must be used for GOD!!

p.s. I am interested also to write to your friend at the University of Nijmegen Andre Geim because it was his idea first, although I cannot give you a million pounds each! When is he in Bradford on Avon, or can I visit him in Holland? Or can you give me his address so I can write to him?

I look forward to your early responses,

Olaf Van Haarve,

The Snakehead.

I put in five pounds for you Professor Main as good faith. Of course I would like a receipt please but you can put in "petrol" or "stationary" or whatever is good for you. This is only the start.

[aus http://www.sci.kun.nl/hfml/levitation-pubres.html]